Sirindhorn International Institute of Technology

Thammasat University at Rangsit

School of Information, Computer and Communication Technology

ECS 455: Problem Set 6

Semester/Year: 2/2016

Course Title: Mobile Communications
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Course Web Site: http://www2.siit.tu.ac.th/prapun/ecs455/

Due date: May 17, 2017 (Wednesday), 4:30 PM

Instructions

1. (1 pt) Write your first name and the last three digits of your student ID on the upper-right corner of every submitted sheet.
2. (1 pt) For each part, write your explanation/derivation and answer in the space provided.
3. (8 pt) It is important that you try to solve all non-optional problems.
4. Late submission will be heavily penalized.

Questions

$n = 7$ $k = 4$

1. Consider a cyclic (7,4) code whose generator polynomial is $x^3 + x + 1$.
   a. Suppose the message is 0100.
      i. Find the corresponding codeword when the code is nonsystematic.

$n=7$. So, $\deg(c(x)) = 6$.

$\underline{m} = 0100 \Leftrightarrow m(x) = x$

$c(x) = m(x)g(x) = x(x^3+x+1) = x^4+x^2+x = 0+1x+1x^2+0x^3+1x^4+0x^5+0x^6$

$\Leftrightarrow \underline{c} = 0110100$

      ii. Find the corresponding codeword when the code is systematic.

$c(x) = x^{n-k}m(x) + r(x)$

$n-k = 7-4 = 3$

$x^{n-k}m(x) = x^3 m(x) = x^4$

$$\begin{array}{r|l} & x \\ \hline x^3+x+1 & x^4 \\ & x^4 + x^2 + x \\ \hline & x^2 + x \end{array}$$

$r(x) = 0+1x+1x^2 \Leftrightarrow 011$

So, $\underline{c} = \boxed{011\underbrace{0100}_{\underline{m}}}$

b. Suppose the message is 1000. Find the corresponding codeword.

i. Find the corresponding codeword when the code is nonsystematic.

$\underline{m} = 1000 \quad \Leftrightarrow m(x) = 1$

$c(x) = m(x)g(x) = 1(x^3 + x + 1) = x^3 + x + 1 = 1 + 1x + 0x^2 + 1x^3 + 0x^4 + 0x^5 + 0x^6$

$\Leftrightarrow \underline{c} = 1101000$

ii. Find the corresponding codeword when the code is systematic.

$c(x) = x^{n-k} m(x) + r(x)$

$n - k = 7 - 4 = 3$

$x^{n-k} m(x) = x\, m(x) = x^3$

$$\begin{array}{r|l} & 1 \\ x^3+x+1 & x^3 \\ & x^3 + x + 1 \\ \hline & x + 1 \end{array}$$

$r(x) = 1 + 1x + 0x^2 \Leftrightarrow 110$

So, $\underline{c} = \boxed{110\underbrace{1000}_{\underline{m}}}$

c. The answers from part (a) and part (b) may be misleading because it turned out that the last four bits of all the codewords found above were exactly the same as the four message bits. So, in this part, we want to show that the non-systematic encoding recipe does give a non-systematic code.
Suppose the message is 0011. Find the corresponding codeword when the code is nonsystematic.

$\underline{m} = 0011 \quad \Leftrightarrow m(x) = 0 + 0x + 1x^2 + 1x^3 = x^2 + x^3$

$c(x) = m(x)g(x) = (x^2 + x^3)(1 + x + x^3) = x^2 + x^3 + x^5 + x^3 + x^4 + x^6 = 0 + 0x + 1x^2 + 0x^3 + 1x^4 + 1x^5 + 1x^6$

$\Leftrightarrow \underline{c} = 0010111$

2. In a (synchronous) CDMA system, suppose there are four users. The codes for user 1 and user 2 are

$\underline{\mathbf{c}}^{(1)} = [-1 \quad -1 \quad -1 \quad 1 \quad 1 \quad -1 \quad 1 \quad 1]$ and

$\underline{\mathbf{c}}^{(2)} = [-1 \quad -1 \quad 1 \quad -1 \quad 1 \quad 1 \quad a \quad -1]$, respectively.

The codes for user 3 and user 4 are unknown.

a. Find a.

All codes in a CDMA system are orthogonal.

$\langle \underline{c}^{(1)}, \underline{c}^{(2)} \rangle = \cancel{1} + \cancel{1} - \cancel{1} - \cancel{1} + \cancel{1} - \cancel{1} + a - 1 \overset{\downarrow}{=} 0$

$\Rightarrow a = 1$

b. At the receiver, suppose we receive $\underline{\mathbf{r}} = [\,-2 \quad 4 \quad 4 \quad 0 \quad 6 \quad 14 \quad -8 \quad -10]$.
Ignore the noise and the fading effect.
Find the message symbols s_1 and s_2 (for user 1 and user 2, respectively).

In class, we showed that $\hat{s}_k = \frac{1}{8}\langle \underline{r}, \underline{c}^{(k)}\rangle$.

$\langle \underline{c}^{(k)}, \underline{c}^{(k)}\rangle = 8$ for $k = 1, 2$.

$$\hat{s}_1 = \frac{1}{8}\langle \underline{r}, \underline{c}^{(1)}\rangle = \frac{1}{8}\left(\cancel{2}+(-4)+(-4)+0+\cancel{6}+(-14)+\cancel{(-8)}+(-10)\right) = -\frac{32}{8} = -4$$

$$\hat{s}_2 = \frac{1}{8}\langle \underline{r}, \underline{c}^{(2)}\rangle = \frac{1}{8}\left(\cancel{2}+\cancel{(-4)}+\cancel{4}+0+\cancel{6}+14+\cancel{(-8)}+10\right) = \frac{24}{8} = 3$$

3. In a (synchronous) CDMA system, suppose there are four users. The codes for user 1 and user 2 are

$\underline{\mathbf{c}}^{(1)} = [23, 69, -23, 46]$ and

$\underline{\mathbf{c}}^{(2)} = [40, -20, 60, a]$, respectively.

The codes for user 3 and user 4 are unknown.

a. Find a.

As in the previous problem, orthogonality implies $\langle \underline{c}^{(1)}, \underline{c}^{(2)}\rangle = 0$.

$$\Rightarrow (23)(40) + (69)(-20) + (-23)(60) + 46a = 0$$
$$920 - 1380 - 1380 + 46a = 0$$
$$46a = 1840$$
$$a = 40$$

b. At the receiver, suppose we receive $\underline{\mathbf{r}} = [759, -553, 131, -692]$.
Ignore the noise and the fading effect.
Find the message symbols s_1 and s_2 (for user 1 and user 2, respectively).

$$\hat{s}_1 = \frac{\langle \underline{r}, \underline{c}^{(1)}\rangle}{\langle \underline{c}^{(1)}, \underline{c}^{(1)}\rangle} = \frac{17457 - 38157 - 3013 - 31832}{529 + 4761 + 529 + 2116} = \frac{-55545}{7935} = -7$$

$$\hat{s}_2 = \frac{\langle \underline{r}, \underline{c}^{(2)}\rangle}{\langle \underline{c}^{(2)}, \underline{c}^{(2)}\rangle} = \frac{30360 + 11060 + 7860 - 27680}{1600 + 400 + 3600 + 1600} = \frac{21600}{7200} = 3$$

4. (Optional) In a CDMA system, each bit time is subdivided into *m* short intervals called **chips**. We will use *m* = 8 chips/bit for simplicity. Each user (MS) is assigned a unique 8-bit code called a **chip-sequence**. To transmit a “1”, a station sends its chip sequence. To transmit a “0”, it sends the one’s complement[1] of its chip sequence.
   Here are the binary chip sequences for four stations:
   A: 0 0 0 1 1 0 1 1
   B: 0 0 1 0 1 1 1 0
   C: 0 1 0 1 1 1 0 0
   D: 0 1 0 0 0 0 1 0
   For pedagogical purposes, we will convert the codes into bipolar form **with binary 0 being -1 and binary 1 being +1**. Under such format, during each bit time, a station can transmit a 1 by sending its chip sequence, it can transmit a 0 by sending the negative of its chip sequence, or it can be silent and transmit nothing. We assume that all stations are synchronized in time, so all chip sequences begin at the same instant.
   When two or more stations transmit simultaneously, their bipolar signals add linearly.
   a. Suppose that A, B, and C are simultaneously transmitting 0 bits. What is the resulting (combined) bipolar chip sequence (in the air)?
   b. Suppose the receiver (BS) gets the following chips: [-1, +1, -3, +1, -1, -3, +1, +1]. Which stations transmitted, and which bits did each one send?
   c. One of your friends wants to work on part (a) and (b) using MATLAB. Here is his code with two incomplete lines.

```
%Chip sequences
C = [0 0 0 1 1 0 1 1; 0 0 1 0 1 1 1 0; 0 1 0 1 1 1 0 0; 0 1 0 0 0 0 1 0];
C = 2*C-1; %Change to bipolar form

% Part a
s = [-1 -1 -1 0] %message to transmit
x = %%%%%%%%%HELP ME%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% Part b
r = [-1 1 -3 1 -1 -3 1 1];
s_hat = 1/8* %%%%%%%%%HELP ME%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%This gives [sA sB sC sD]' in bipolar form;
%The value is 1 if 1 was transmitted. The value is 0 if nothing was
%transmitted. The value is -1 if 0 was transmitted.
```

   Help him find the expression for “`s`” and “`s_hat`” in the code above.

   Note that the expression for `x` should be in terms of `C` and `s`. The expression for `s_hat` should be in terms of `C` and `r`.

[1] You should have seen the “one’s complement” operation in your “digital circuits” class.

Solution

Because the script for part (c) already solves part (a) and (b), we will work on part (c) first.

(c) Use the above MATLAB code with `x = s*C`; and `s_hat = (C*r')/8`;

(a) [3   1 1 -1 -3 -1 -1 1]

(b) [1 -1 0 1]'; Hence, A and D sent 1 bits, B sent a 0 bit, and C was silent.